3.4 Communicate With Algebra

You have seen how algebra tiles can be used to model algebraic expressions.

The model below shows that Petra mowed a square lawn of unknown area once a week for 4 weeks.

The algebraic representation is $4x^2$. Look at each part of this expression. What does the 4 represent? What does the x^2 represent?

term

- an expression formed by the product of numbers and/or variables

The expression $4x^2$ is called a **term**. A term consists of two parts:

When you represent an algebraic expression using algebra tiles, the variable in the expression tells you which type of tile to use. To represent x^2, use an x^2-tile.

x^2-tile

The number of tiles corresponds to the coefficient. Since the expression is $4x^2$, there are four x^2-tiles.

Example 1 Identify Coefficients and Variables

Identify the coefficient and the variable of each term.

a) Jim earns \$7 per hour at his part-time job. If he works for x hours, his earnings, in dollars, are $7x$.

b) The depth, in metres, of a falling stone in a well after t seconds is $-4.9t^2$.

c) The area of a triangle with base b and height h is $\frac{1}{2}bh$.

d) The area of a square with side length k is k^2.

e) Amir walks 6 blocks to school.

Solution

	Expression	Coefficient	Variable	Comments
a)	$7x$	7	x	
b)	$-4.9t^2$	-4.9	t^2	The negative sign is included with the coefficient.
c)	$\frac{1}{2}bh$	$\frac{1}{2}$	bh	The variable can consist of more than one letter or symbol.
d)	k^2	1	k^2	When the coefficient is not shown, it is 1.
e)	6	6	none	A term with no variable is called a **constant term**, or simply a **constant**.

Literacy Connections

The **coefficient** is also called the **numerical coefficient**. It is a number only.

The **variable** is also called the **literal coefficient**. It consists of one or more variables and their exponents, if they exist.

Exponents on the variables belong to the literal coefficient, because they represent a product of variables: $x^2 = x \times x$.

A **polynomial** can be classified by the number of terms it has.

Type of Polynomial	Number of Terms	Examples
monomial	1	x, $3y$, $-4a^2$, 5
binomial	2	$2x - 3$, $ab + 2a$, $0.4x^2 - x$
trinomial	3	$2x^2 + 3x - 1$, $a + 2b - c$

polynomial

- an algebraic expression consisting of one or more terms connected by addition or subtraction operators

Literacy Connections

The prefixes of the polynomial names have the following meanings:

- *mono* means 1
- *bi* means 2
- *tri* means 3

Example 2 Classify Polynomials by Name

Classify each polynomial by the number of terms it has.

a) $3x^2 + 2x$ **b)** $-2m$ **c)** $4x^2 - 3xy + y^2$ **d)** $a - 2b + c - 3$

Solution

	Polynomial	Number of Terms	Type of Polynomial
a)	$3x^2 + 2x$	2	binomial
b)	$-2m$	1	monomial
c)	$4x^2 - 3xy + y^2$	3	trinomial
d)	$a - 2b + c - 3$	4	four-term polynomial

$3x^2 + 2x$ has two terms, $3x^2$ and $2x$.

I can find the number of terms by looking for the addition and subtraction operators that separate the terms:

$4x^2 - 3xy + y^2$

Two operators separate the three terms in this trinomial.

Example 3 Classify Terms by Degree

degree of a term

- the sum of the exponents on the variables in a term

Find the **degree of each term**.

a) x^2 **b)** $3y^4$ **c)** $0.7u$

d) $-2a^2b$ **e)** $\frac{2}{3}xy$ **f)** -5

Solution

Look at the exponents of the variables. Add them if there is more than one.

Remember, when no exponent appears on a variable, the value of the exponent is 1. For example, $4u = 4u^1$.

If the term has no variable at all, then the degree is 0.

	Term	Sum of Exponents	Degree
a)	x^2	2	2
b)	$3y^4$	4	4
c)	$0.7u^1$	1	1
d)	$-2a^2b^1$	$2 + 1 = 3$	3
e)	$\frac{2}{3}x^1y^1$	$1 + 1 = 2$	2
f)	-5	0	0

Example 4 Classify Polynomials by Degree

degree of a polynomial

- the degree of the highest-degree term

Find the **degree of each polynomial**.

a) $x + 3$

b) $5x^2 - 2x$

c) $3y^3 + 0.2y - 1$

d) $7x^2y^4 + x^6y$

Solution

The degree of the first term is 1. The degree of the second term is 0. The highest degree is 1.

The degree of the first term is $2 + 4$. The degree of the second term is $6 + 1$.

	Polynomial	Term With Highest Degree	Degree of Term in Column 2	Degree of Polynomial
a)	$x + 3$	x	1	first
b)	$5x^2 - 2x$	$5x^2$	2	second
c)	$3y^3 + 0.2y - 1$	$3y^3$	3	third
d)	$7x^2y^4 + x^6y$	x^6y	7	seventh

Example 5 Use an Algebraic Model to Solve a Problem

Cheryl works part-time as a ski instructor. She earns $125 for the season, plus $20 for each children's lesson and $30 for each adult lesson that she gives.

a) Write an expression that describes Cheryl's total earnings for the season. Identify the variable and the coefficient of each term and explain what they mean.

b) One winter, Cheryl gave eight children's lessons and six adult lessons. What were her total earnings?

Solution

a) Cheryl's total earnings can be described by the polynomial expression $20c + 30a + 125$.

Term	Variable	Meaning of Coefficient
$20c$	c represents the number of children's lessons.	20 represents the earnings per children's lesson.
$30a$	a represents the number of adult lessons.	30 represents the earnings per adult lesson.
125	There is no variable.	Cheryl has fixed earnings of $125 per season.

b) Substitute $c = 8$ and $a = 6$, and evaluate the expression.

$$\begin{aligned} & 20c + 30a + 125 \\ = \ & 20(8) + 30(6) + 125 \\ = \ & 160 + 180 + 125 \\ = \ & 465 \end{aligned}$$

Cheryl's total earnings for this season were $465.

Key Concepts

- Algebraic expressions can be used to communicate mathematical ideas.
- A term is the product of a coefficient and variable part.
- A polynomial can be a single term or a combination of terms using addition or subtraction operators.
- A polynomials can be classified
  - by the number of terms it has
  - by its degree

Communicate Your Understanding

C1 Create two examples of each.

a) monomial **b)** binomial

c) trinomial **d)** four-term polynomial

C2 Julio says that the term x^2 has a coefficient of 2 and a variable x. Is Julio correct? Explain.

C3 **a)** Are these expressions equivalent? Explain.
$2w + 1t$ $2w + t$

b) Are these expressions equivalent? Explain.
$3x + 1$ $3x$

c) Explain when you must write the number 1, and when you do not need to.

Practise

For help with question 1, see Example 1.

1. Identify the coefficient and the variable part of each term.

a) $2y$ **b)** $-3x$ **c)** mn

d) $\frac{1}{2}x^2$ **e)** $-w^2$ **f)** $-0.4gh^3$

For help with questions 2 and 3, see Example 2.

2. $7x^2 + 3xy + 4y^2$ is a

A monomial **B** binomial

C trinomial **D** term

3. Classify each polynomial by the number of terms.

a) $-2x$ **b)** $6y^2 + 2y - 1$

c) $a - \frac{1}{2}b$ **d)** $3u^2 - uv + 2v^2$

e) $3k^2 - \frac{1}{2}k$ **f)** $m + 0.2n - 0.3 + mn$

For help with questions 4 and 5, see Example 3.

4. The degree of $4u - 5u^2 + 9$ is

A 1 **B** 2 **C** 3 **D** 0

5. State the degree of each term.

a) $5x^2$ **b)** $-6y$ **c)** -3

d) u^2v^4 **e)** $\frac{1}{3}x^2y^3$ **f)** $0.2a^2b$

For help with question 6, see Example 4.

6. State the degree of each polynomial.

a) $3x - 4$ **b)** $y^2 + 3y - 1$ **c)** $m - 2m^3$

d) $a^3b^2 - 8a^2b^5$ **e)** $2x^2y^4 - \frac{2}{5}xy^3$

For help with questions 7 and 8, see Example 4.

7. In a TV trivia show, a contestant receives 500 points for a correct answer and loses 200 points for an incorrect answer. Let c represent the number of correct answers and i represent the number of incorrect answers. Which expression describes a contestant's total points?

A $500c + 200i$ **B** $500c - 200i$

C $500i + 200c$ **D** $500i - 200c$

8. A hockey team earns 2 points for a win and 1 point for a tie. Let w represent the number of wins and t represent the number of ties.

a) Which expression can be used to describe the total points?

A $2w + 1$ **B** $w + t$ **C** $2w + 1t$ **D** $2w + t$

b) Is there more than one correct answer? Justify your answer.

9. Substitute the given values and evaluate each expression.

a) $3x + 5$ $x = 2$

b) $4y + 4$ $y = -2$

c) $a^2 + 2b - 7$ $a = 4, b = 1$

d) $2m^2 - 3n + 8$ $m = -2, n = 5$

Connect and Apply

10. The students at Prince Albert Public School sell magazine subscriptions to raise money. The school receives 37% of the money paid for the subscriptions.

a) Choose a variable to represent the money paid for the subscriptions.

b) Using your variable from part a), write an expression for the amount of money the school will receive.

c) Tyler sold one magazine subscription to his aunt for $25.99. Calculate the amount the school receives on this sale.

d) The sum of all the subscription orders was $4257.49. Use your formula to calculate how much the school will receive for this fundraiser.

11. Meredith has a summary job at a fitness club. She earns a $5 bonus for each student membership and a $7 bonus for each adult membership she sells.

a) Write a polynomial expression that describes Meredith's total bonus.

b) Identify the variable and the coefficient of each term and explain what they mean.

c) How much will Meredith's bonus be if she sells 12 student memberships and 10 adult memberships?

12. An arena charges $25 for gold seats, $18 for red seats, and $15 for blue seats.

a) Write an expression that describes the total earnings from seat sales.

b) Identify the variable and the coefficient of each term and explain what they mean.

c) How much will the arena earn if it sells 100 gold seats, 200 blue seats, and 250 red seats?

13. On a multiple-choice test, you earn 2 points for each correct answer and lose 1 point for each incorrect answer.

a) Write an expression for a student's total score.

b) Maria answered 15 questions correctly and 3 incorrectly. Find Maria's total score.

14. a) Describe a situation that can be modelled by an algebraic expression.

b) Select variables and write the expression.

c) Illustrate your expression using algebra tiles or a diagram.

15. Write a response to this e-mail from a classmate.

FROM: ManuelS
TO: JillP
SUBJECT: changing a decimal to a percent

Hi Jill,

Too bad I missed today's class. I copied your notes and I get most of it, I guess. I'm just a little confused about the difference between a term and a polynomial. Can you please help me out?

Thanks,
Manuel

16. Chapter Problem Alysia is designing a logo for her school team, the Eagles. The design will be used to make different-sized crests for clothing such as jackets, sweaters, and baseball caps. How can Alysia make sure that, when the crest is made larger or smaller, the shape will not change?

The height will always be double the width.

a) If w represents the width, what expression represents the height?

b) How high will a crest that is 5 cm wide be?

c) How wide will a crest that is 25 cm high be?

Achievement Check

17. In a soccer league, teams receive 3 points for a win, 2 points for a loss, and 1 point for a tie.

a) Write an algebraic expression to represent a team's total points.

b) What variables did you choose? Identify what each variable represents.

c) The Falcons' record for the season was 5 wins, 2 losses, and 3 ties. Use your expression to find the Falcons' total points.

d) The 10-game season ended with the Falcons tied for second place with the same number of points as the Eagles. The Eagles had a different record than the Falcons. How is this possible?

Extend

18. Alberto is training for a triathlon, where athletes swim, cycle, and run. During his training program, he has found that he can swim at 1.2 km/h, cycle at 25 km/h, and run at 10 km/h. To estimate his time for an upcoming race, Alberto rearranges the formula distance = speed × time to find that: time = $\frac{\text{distance}}{\text{speed}}$.

a) Choose a variable to represent the distance travelled for each part of the race. For example, choose s for the swim.

b) Copy and complete the table. The first row is done for you.

Part of the Race	Speed (km/h)	Distance (km)	Time (h)
swim	1.2	s	$\frac{s}{1.2}$
cycle			
run			

c) Write a trinomial to model Alberto's total time.

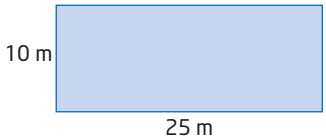
d) A triathlon is advertised in Kingston. Participants have to swim 1.5 km, cycle 40 km, and run 10 km. Using your expression from part c), calculate how long it will take Alberto to finish the race.

e) Is your answer a reasonable estimate of Alberto's triathlon time? Explain.

I can use the formula

$\text{time} = \frac{\text{distance}}{\text{speed}}$ to calculate Ashleigh's travel time.

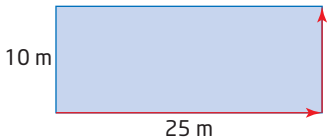
19. Ashleigh can walk 2 m/s and swim 1 m/s. What is the quickest way for Ashleigh to get from one corner of her pool to the opposite corner?



a) Predict whether it is faster for Ashleigh to walk or swim.

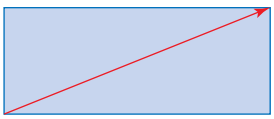
b) Ashleigh can walk at a speed of 2 m/s. The time, in seconds, for Ashleigh to walk is $\frac{w}{2}$, where w is the distance, in metres, she walks. Use this relationship to find the travel time if Ashleigh walks around the pool.

Path 1: Walk the entire distance.



c) Write a similar expression to represent the time taken for Ashleigh to swim a distance s. Her swimming speed is 1 m/s. Use this relationship to find the travel time if Ashleigh swims straight across.

Path 2: Swim the entire distance.

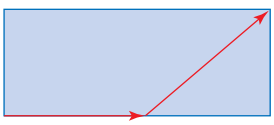


d) Which route is faster, and by how much?

20. Refer to question 19.

a) Do you think it will be faster for Ashleigh to walk half the length and then swim? Explain your reasoning.

Path 3: Walk half the length, then swim.



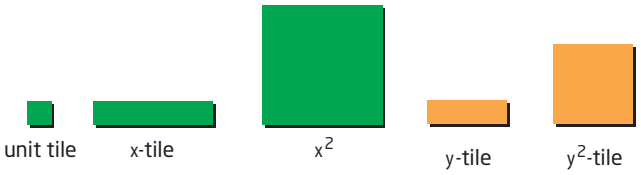
b) Find the travel time for this path. Compare this with your answers to question 19.

c) Do you think this is the fastest possible path? Find the fastest path and the minimum time required to cross the pool, corner to opposite corner. Describe how you solved this.

Did You Know?

Sixteen year-old Marilyn Bell of Toronto was the first person to swim across Lake Ontario. In September 1954 Marilyn swam from Youngstown, NY, to the CNE grounds in Toronto. The 51.5 km distance took her 21 h.

21. Some algebraic expressions involve more than one variable. You can model these using an expanded set of algebra tiles.



Fly By Night Aero Insurance company charges \$500 for liability, plus 10% of the value of the plane, plus \$300 per seat. Let v represent the value of the plane and s represent the number of seats. The cost of the insurance is modelled by $C = 500 + 0.1v + 300s$. A four-seat Piper Cherokee valued at \$30 000 would cost $500 + 0.1(30\,000) + 300(4)$ or \$4700 per year to insure.

a) Explain how you would use tiles to represent $0.1v$.

b) Explain how you would use tiles to represent $300s$.

c) Build an algebraic model to describe the cost for airplane insurance for the Piper Cherokee, using algebra tiles, diagrams, or virtual algebra tiles.

d) Find the cost of insurance for a 50-seat plane, valued at \$500 000.

22. Math Contest If $3m + 5 = 23$ and $2n - 7 = 21$, the value of $3m + 2n$ is

A 20 **B** 44 **C** 46 **D** 54 **E** 56

23. Math Contest If $a^x \times b^y \times c^z = 18\,144$ and a, b, and c are all prime numbers, the value of $x + y + z$ is

A 7 **B** 10 **C** 11 **D** 12 **E** 20

24. Math Contest Find value(s) of m for which $\left(\frac{1}{2}\right)^{2m} = \left(\frac{1}{4}\right)^{m}$. Is there more than one possible value of m? Explain.

Virtual Algebra Tiles With *The Geometer's Sketchpad®*

You can create and manipulate virtual algebra tiles using computer software such as *The Geometer's Sketchpad®*. You can build algebraic models, plus you can change the length of the variable tiles.

Investigate

Tools

- computer equipped with *The Geometer's Sketchpad®*
- **Algebra Tiles.gsp**

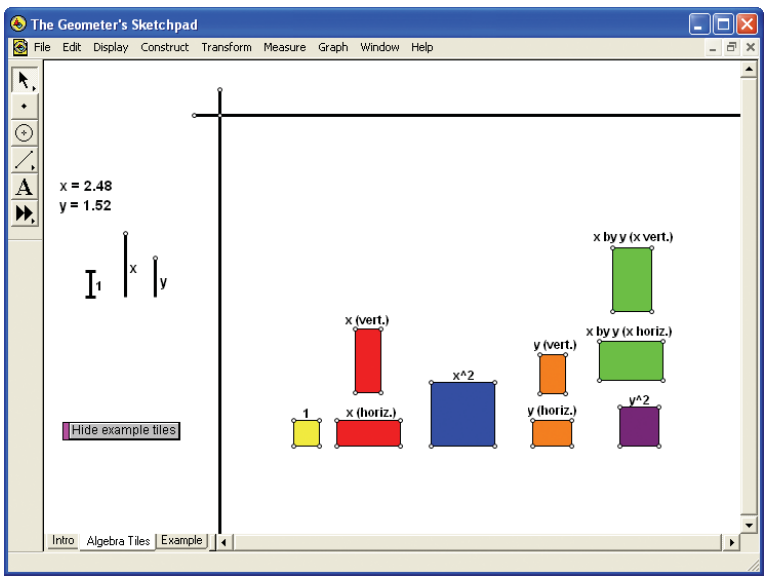
How can you build algebraic models using virtual algebra tiles?

1. a) Start *The Geometer's Sketchpad®* and open the sketch **Algebra Tiles.gsp**.

b) Read the instructions and click on the **Algebra Tiles page** button.

2. Explore the pre-made tiles.

a) Click on the **Show example tiles** button.



b) Click and drag the top of the x-slider. Which tiles are affected by the x-slider?

c) Repeat part b) for the y-slider.

d) The unit tile seems to have a slider next to the x-slider. Try to change its length, and describe what happens. Why can you not change the dimensions of the unit tile?

For the rest of this activity, you will work only with

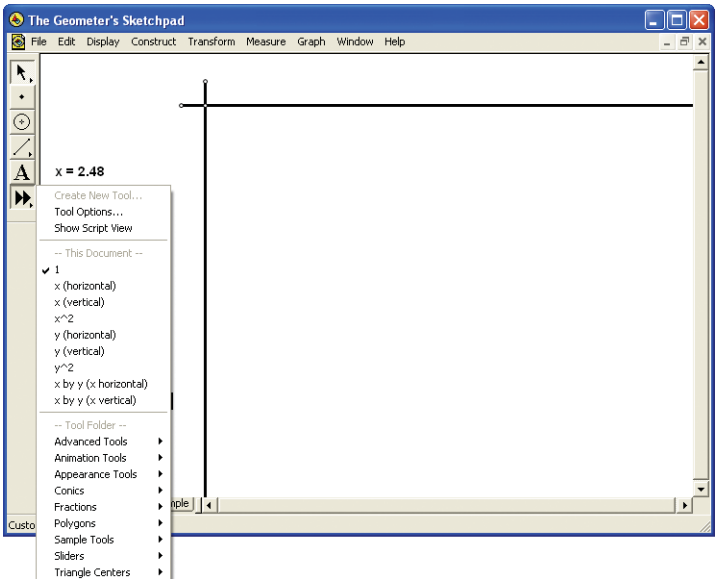
- unit tiles 

- x-tiles (horizontal and vertical) 

- x^2-tiles 

3. Clear the workspace and bring out only the algebra tiles you need:

- Click on **Hide example tiles**.
- Click and hold the **Custom Tool** icon at the left side of the screen.



- Select **1** (the unit tile) and place a unit tile somewhere on the workspace.
- Click and hold the **Custom Tool** again and select **x (horizontal)** and place a tile on the workspace.
- Repeat for the **x (vertical)** and **x^2** (x^2) tiles.

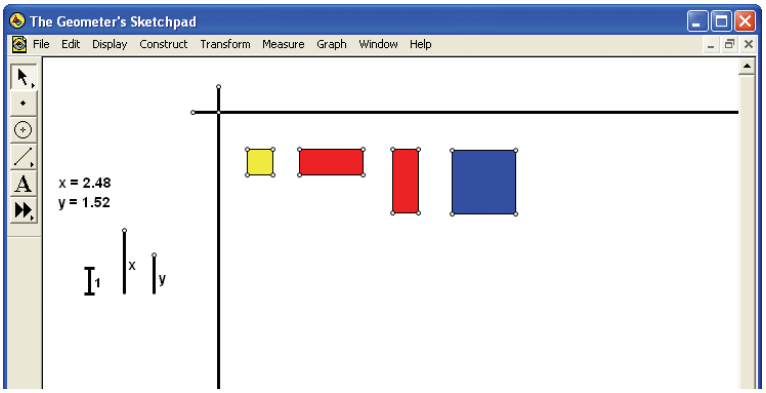
Technology Tip

If you do not see the list of tools when you click on the **Custom Tool** icon, try holding the left mouse button for a couple of seconds, and they will appear.

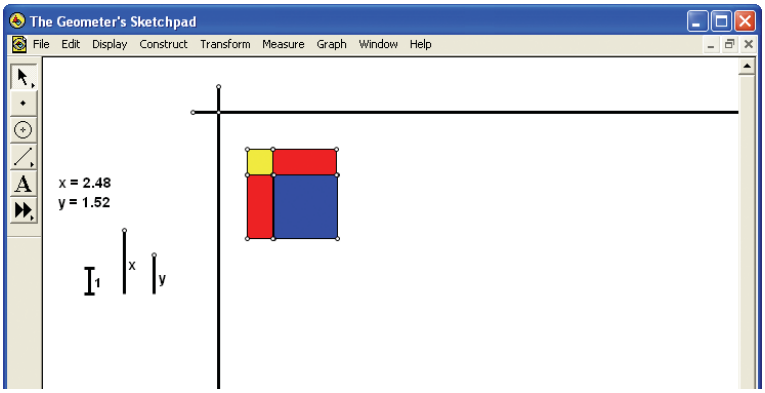
Technology Tip

You can add more tiles by using the **Custom Tool** or by using the **Copy** and **Paste** commands in the **Edit** menu. To copy an entire tile, use the **Selection Arrow Tool** to click and drag a dashed box around the tile you want to copy. Then, paste it and move it wherever you like.

Once you have all four tiles, choose the **Selection Arrow Tool** and click somewhere in the white space to deselect the last object you created.



4. Explore the relationships between the tiles. Arrange the tiles as shown, by clicking and dragging them one at a time.



a) Move the x-slider and describe what happens to the length and width of each tile.

Tile	Length (changes/does not change)	Width (changes/does not change)
unit		
x (horizontal)		
x (vertical)		
x^2		

b) Describe how the length of the x-tile is related to the length and width of the x^2-tile. Why is this so?

c) Describe how the width of the x-tile is related to the length and width of the unit tile. Why is this so?

5. a) Why is it that you can change the length of an x-tile, but not a unit tile?

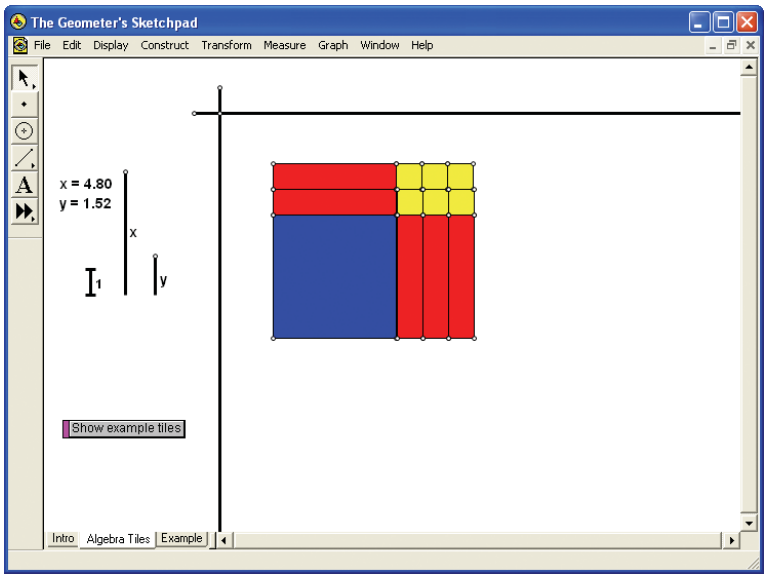
b) What advantage does this give to virtual algebra tiles over physical algebra tiles?

6. Build each algebraic model using virtual algebra tiles.

a) $3x + 2$ **b)** $x^2 + 2x + 5$

7. Build each algebraic model and try to put the parts together to form a rectangle. You may have to change horizontal x-tiles to vertical ones, or vice versa. Once you have built the rectangle, write expressions for its length and width. The first one is shown as an example.

a) $x^2 + 5x + 6$



The length is $x + 3$ and the width is $x + 2$.

Now try these.

b) $x^2 + 2x + 1$

c) $x^2 + 7x + 12$

8. a) Create your own example like the ones in step 7. Not all expressions will work. Make sure that you can create a rectangle to model your expression.

b) Trade expressions with a classmate and try to build each other's algebraic models.

9. Describe at least one advantage and one disadvantage of using virtual algebra tiles to build algebraic models.